RESEARCH WORKSHOP OF ISRAEL SCIENCE FOUNDATION ALGEBRAIC MODES OF REPRESENTATIONS AND NILPOTENT ORBITS: THE CANICULAR DAYS.
CELEBRATING A. JOSEPH'S 75 BIRTHDAY

19-23 JULY 2017

Abstracts

Christopher Bowman

Title: Complex reflection groups of type $G(l, 1, n)$ and their deformations.

Abstract: We explore a new approach to understanding the modular representations of Hecke algebras of complex reflection groups, this makes heavy use of Websters theory of diagrammatic Cherednik algebras. We discuss a super strong linkage principle which provides degree-wise upper bounds for graded decomposition numbers (this is new even in the case of symmetric and general linear groups). We also consider higher-level analogues of homomorphisms between Weyl/Specht modules which are generically placed (within the associated alcove geometries) and the classical Lusztig conjecture (over fields of sufficiently large characteristic).

Amiram Braun

Title: On Humphreys "parameterization of blocks" conjecture.

Abstract: Let G be a connected, semi-simple, simply connected algebraic group over an algebraically closed field of prime characteristic p. Let $\mathfrak{g} = \mathrm{Lie}(G)$ and $U(\mathfrak{g})$ its enveloping algebra. We shall explain the above 1998 conjecture about blocks of $U(\mathfrak{g})$,discuss the known cases (Brown-Gordon 2001) and sketch our proof of it in case "p is good for G". We shall also describe some of the new results about $U(\mathfrak{g})$ which are crucial in this proof.

Giovanna Carnovale

Title: Normality of quotients of closures of Jordan classes.

Abstract: Let G be a complex reductive algebraic group and let $\mathfrak{g}$ be its Lie algebra. Jordan classes (or decomposition classes) in $\mathfrak{g}$ were introduced by Borho and Kraft. They are unions of adjoint orbits that are all isomorphic as homogenous spaces. Their closures form a stratification that is very well suited for understanding, for instance, the sheets of $\mathfrak{g}$ for the adjoint action. Borho has described the normalization of the geometric quotients of these closures and Richardson, Broer, Douglass and Rohrle have listed the normal ones.

A group analogue for Jordan classes has been introduced by Lusztig in the costruction of generalized Springer correspondence and it has been used by Esposito and the speaker in the description of the sheets in G for the conjugation action. In analogy to Borho's result, the normalisation of the geometric quotient of their closures is easily described. In this talk I will describe how to obtain the full list of the geometric quotients of closures of Jordan classes that are normal.

This is a joint work with Francesco Esposito.

Alexander Elashvili

Title: On maximal commuting subspaces related to nilpotents in simple Lie algebras.

Abstract: Let e be a nilpotent element of a simple Lie algebra $\mathfrak{g}$, defined over an algebraically closed field k with $\operatorname{char} k = 0$. Embed e into the subalgebra $\mathfrak{sl}(2,k) = \langle f, h, e\rangle$. Then h defines a $\mathbb{Z}$-grading $\mathfrak{g} = \sum_{i\in\mathbb{Z}} \mathfrak{g}(i)$, where $\mathfrak{g}(i) = \{x \in \mathfrak{g} \mid [h, x] = ix\}$.

In the talk, we will give complete answer to the following question: for which nilpotents e does there exist in $\mathfrak{g}(1)$ a commutative subspace C with $\dim C = \frac{1}{2} \dim \mathfrak{g}(1)$? More generally, we will discuss the structure of maximal commutative subspaces in $\mathfrak{g}(1)$ for arbitrary e.

This is a joint work in progress with M. Jibladze and V. Kac.

Inna Entova-Aizenbud

Title: Deligne categories and complexes of representations of symmetric groups.

Abstract: Let V be a finite-dimensional (complex) vector space, and $Sym(V)$ be the symmetric algebra on this vector space. We can consider the multiplication map $Sym(V) \otimes V \to V$ as a complex of $GL(V)$-representations of length 2.

I this talk, I will describe how tensor powers of the above complex define interesting complexes of representations of the symmetric group S_n, which were studied by Deligne in the paper "La Categorie des Representations du Groupe Symetrique S_t, lorsque t nest pas un Entier Naturel".

I will then explain how computing the cohomology of these complexes helps establish a relation between the Deligne categories and the representations of S_∞, which are two natural settings for studying stabilization in the theory of finite-dimensional representations of the symmetric groups.

Florence Fauquant-Millet

Title: About polynomiality of the Poisson-semicentre for parabolic subalgebras

Abstract: In my lecture I will review some results on polynomiality of the Poisson-semicentre for parabolic subalgebras which I got with (or inspired by) Anthony Joseph.

Especially when we consider a simple Lie algebra $\mathfrak{g}$ of type A or C, one of our important results is that the Poisson-semicentre of the symmetric algebra of any parabolic (even biparabolic) subalgebra $\mathfrak{p}$ of $\mathfrak{g}$ is a polynomial algebra, for which the number of generators and the weights for each of them can be computed.

Actually a combinatorial (but not necessary) criterion says for which parabolic subalgebras $\mathfrak{p}$ of a simple Lie algebra $\mathfrak{g}$ of type outside A and C, the Poisson-semicentre is a polynomial algebra.

When the simple Lie algebra $\mathfrak{g}$ is of type B or D and when $\mathfrak{p}$ is a maximal parabolic subalgebra of $\mathfrak{g}$, the criterion applies roughly for half of the cases.

For the rest of the cases, Polyxeni Lamprou and myself have constructed adapted pairs (an adapted pair - introduced by Anthony Joseph in 2007 - may be viewed as an analogue of a principal $\mathfrak{sl}_2$-triple for a non-reductive Lie algebra). Then a refinement of the above criterion allows us to conclude on the polynomiality of the Poisson-semicentre.

Yasmine Fittouhi

Title: Algebraic and geometrical description of the fibers of the Mumford system.

Abstract: Mumford system is an algebraically completely integrable system (a.c.i). Our main focus is to study the fibers of the momentum map, We distinguish two types of fibers: smooth fibers described geometrically as well as an algebraically by Mumford, and singular fibers. For the purpose to describe the singular fibers, one needs to use the concept of stratification where each stratum is determined by the behaviour of the vectors fields of the Mumford system. The previous step would permit us to deduce an algebraic characteristic of each stratum which allows us to relate them with fiber of Mumford system of leaser degrees. To conclude the study, we prove that a stratum of fiber is isomorphic to an affine part of the generalized Jacobian of hyperelliptic curve.

Lucas Fresse

Title: On the existence of smooth orbital varieties.

Abstract: Let G be a complex reductive group. The orbital varieties are certain Lagrangian subvarieties of the nilpotent G-orbits. They were used by Anthony Joseph for obtaining a geometric construction of Weyl group representations. These varieties are in general singular. In this talk we show that, in the case where G is classical, every nilpotent G-orbit contains at least one smooth orbital variety. We also point out that this property may fail in the case where G is of exceptional type.

This is a joint work with Anna Melnikov.

Crystal Hoyt

Title: A new category of $\mathfrak{sl}(\infty)$-modules related to Lie superalgebras.

Abstract: The (reduced) Grothendieck group of the category of finite-dimensional representations of the Lie superalgebra gl(m—n) is an sl(infinity)-module with the action defined via translation functors, as shown by Brundan and Stroppel. This module is indecomposable and integrable, but does not lie in the tensor category, in other words, it is not a subquotient of the tensor algebra generated by finitely many copies of the natural and connatural sl(infinity)-modules. In this talk, we will introduce a new category of sl(infinity)-modules in which this module is injective, and describe the socle filtration of this module.

This is a joint work with I. Penkov, V. Serganova

Mikhail Ignatyev

Title: Kostant cascades and centrally generated primitive ideals of $U(\mathfrak{n})$ in the orthogonal type.

Abstract: Let $\mathfrak{g}$ be a classical finite-dimensional complex Lie algebra, $\mathfrak{b}$ be a Borel subalgebra of $\mathfrak{g}$, and $\mathfrak{n}$ be the nilradical of $\mathfrak{b}$. The Dixmier map $\mathfrak{n}^* \ni f \mapsto J(f) \in U(\mathfrak{n})$ provides a one-to-one correspondence between the set of primitive ideals of the enveloping algebra $U(\mathfrak{n})$ and the set of coadjoint $\exp \mathfrak{n}$-orbits on the dual space $\mathfrak{n}^*$. Our goal is to describe centrally generated primitive ideals of $U(\mathfrak{n})$ in the case when $\mathfrak{g} = \mathfrak{so}_{2n+1}(\mathbb{C})$, $\mathfrak{so}_{2n}(\mathbb{C})$ or $\mathfrak{so}_\infty(\mathbb{C})$.

In the finite-dimensional case, denote by Φ^+ the set of positive roots of the algebra $\mathfrak{g}$ with respect to $\mathfrak{b}$, and by $\mathcal{B}$ the Kostant cascade of Φ. For example, if $\mathfrak{g} = \mathfrak{so}_{2n}(\mathbb{C})$, then $\mathcal{B} = \{\varepsilon_1 + \varepsilon_2,\ \varepsilon_1 - \varepsilon_2,\ \varepsilon_3 + \varepsilon_4,\ \varepsilon_3 - \varepsilon_4,\ \ldots\}$. Let $\{e_\alpha,\ \alpha \in \Phi^+\}$ be a basis of $\mathfrak{n}$ consisting of root vectors.

Definition. We say that $f \in \mathfrak{n}^*$ is a *Kostant form* if $f(e_\alpha) = 0$ for $\alpha \notin \mathcal{B}$ and $f(e_\alpha) \neq 0$ for $\alpha \in \mathcal{B} \setminus \Delta$, where $\Delta \subset \Phi^+$ is the set of simple roots.

Let J be a primitive ideal of $U(\mathfrak{n})$, and $\Delta_\beta,\ \beta \in \mathcal{B}$, be the set of canonical generators of the center $Z(\mathfrak{n})$ of $U(\mathfrak{n})$ described by J. Dixmier, B. Kostant and A. Joseph. Since J is the annihilator of a simple $\mathfrak{n}$-module, given $\beta \in \mathcal{B}$, there exists the unique $c_\beta \in \mathbb{C}$ such that $\Delta_\beta - c_\beta \in J$. Our main result is as follows.

Theorem. *The following conditions are equivalent*:

i) *J is centrally generated*;

ii) *all scalars c_β, $\beta \in \mathcal{B} \setminus \Delta$, are nonzero*;

iii) *$J = J(f)$ for a Kostant form f.*

We also prove similar results for the simple infinite-dimensional finitary orthogonal Lie algebra $\mathfrak{g} = \mathfrak{so}_\infty(\mathbb{C})$ and some special choice of $\mathfrak{n}$. Note that these results for the special linear algebra and the symplectic algebra were proved earlier by I. Penkov and myself, both in finite-dimensional and infinite-dimensional settings.

Anthony Joseph

Title: S-graphs and identities in Demazure Modules

Abstract: The lecture concerns the structure of the Kashiwara crystal pertaining to a Verma module. In this, the description of dual Kashiwara functions for this crystal leads to the notion of an S-graph. A set of canonical S-graphs are obtained through the decomposition of the graph of links in equivalence classes of tableaux with boundary conditions. It is shown that these canonical S-graphs arise from identities in Demazure modules. Moreover all trails in such modules are described (up to a conjecture asserting the absence of false trails) and these can be used to give a precise description of the Kashiwara crystal.

The lecture is dedicated to the memory of my grand-daughter Bar Sagi who died in February 2017 after a long struggle with bone cancer.

Sefi Ladkani

Title: On the class $\mathcal{P}$ of quivers

Abstract: The class $\mathcal{P}$ of quivers was introduced by Kontsevich and Soibelman in relation with their study of Donaldson-Thomas invariants. It is defined recursively using the operations of triangular extensions and Fomin-Zelevinsky mutations. Two interesting questions thus arise:

— Which quivers are actually in the class $\mathcal{P}$?
— What are the implications of being in class $\mathcal{P}$ on the associated cluster algebras and the possible potentials (in the sense of Derksen-Weyman-Zelevinsky) on a quiver?

Concerning the first question, we will describe all the quivers of finite mutation type that belong to the class $\mathcal{P}$. This builds on results of Felikson-Shapiro-Tumarkin as well as on the theory of cluster algebras arising from marked surfaces by Fomin-Shapiro-Thurston.

We will then present several results partially answering the second question. Finally, we will construct a family of quivers of finite mutation type not belonging to the class $\mathcal{P}$ whose associated cluster algebras and potentials exhibit various pathologies.

Lenny Makar-Limanov

Title: Jacobian conjecture. Some old and new results

Abstract: I am going to talk, as it became customary lately on the Jacobian conjecture. In my talk I'll tell about some old and new results related to this conjecture.

Monty McGovern

Title: Closures of $O(n)$-orbits in the flag variety for $GL(n)$.

Abstract: Abstract: We characterize the $O(2n)$-orbits in the flag variety for $GL(2n)$ with rationally smooth closure via a graph-theoretic criterion. We also give a necessary and sufficient pattern avoidance criterion for rational smoothness, conjecturing that it holds for $O(m)$-orbits in the flag variety for $GL(m)$ with m odd as well.

Dmitri Panyushev

Title: Semi-direct products of Lie algebras and covariants

Abstract:

Let Q be a connected algebraic group with Lie algebra $\mathfrak{q}$. Symmetric invariants of $\mathfrak{q}$, i.e., the ring of Q-invariants in the symmetric algebra $S(\mathfrak{q})$), can be considered as a first approximation to the understanding of the coadjoint representation of Q and coadjoint orbits. In my talk, I consider a class of non-reductive Lie algebras $\mathfrak{q}$, where the description of the symmetric invariants is rather explicit and the coadjoint representation has a number of nice invariant-theoretic properties. If G is a semisimple group with Lie algebra $\mathfrak{g}$ and V is G-module, then $\mathfrak{q}$ is defined to be the semi-direct product of $\mathfrak{g}$ and V. Then one is interested in the case, where the generic isotropy group for the G-action on V is reductive and commutative. Then the symmetric invariants of $\mathfrak{q}$ can be constructed via certain G-equivariant maps from $\mathfrak{g}$ to V ("covariants").

This is a joint work with Oksana Yakimova.

Elena Poletaeva

Title: On finite W-algebras for Lie superalgebras

Abstract: A finite W-algebra is a certain associative algebra attached to a pair $(\mathfrak{g}, e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra or a classical Lie superalgebra and $e \in \mathfrak{g}$ is an even nilpotent element. It is a generalization of the universal enveloping algebra $U(\mathfrak{g})$.

E. Ragoucy and P. Sorba described the finite W-algebras for $\mathfrak{gl}(n)$ when the matrix e has Jordan blocks of the same size. J. Brundan and A. Kleshchev generalized their result to an arbitrary nilpotent element. J. Brown, J. Brundan and S. Goodwin described finite W-algebras for $\mathfrak{gl}(m|n)$ associated to even regular nilpotent e as truncations of shifted super-Yangians of $\mathfrak{gl}(1|1)$.

We study the finite W-algebra for the queer Lie superalgebra $Q(n)$ associated with even nilpotent coadjoint orbits, in the case when the corresponding nilpotent element has Jordan blocks each of size l. We prove that this finite W-algebra is isomorphic to a quotient of the super-Yangian of $Q(\frac{n}{l})$.

This is a joint work with V. Serganova.

Shifi Reif

Title: The Duflo-Serganova functor and character rings of Lie superalgebras.

Abstract: Given an odd element x in a Lie superalgebra, $[x, x]$ is an even element. If this even element is zero, x^2 acts as zero on every module M. The Duflo-Serganova functor sends a module M to $ker_M(x)/xM$. In fact, the image of M under the functor is a module over a Lie superalgebra of smaller rank that shares some properties with the module M, e.g. its superdimension.

In this talk, we will show that even though this functor is not exact, it still induces a homomorphism on the ring of characters. We shall give a basis to the kernel of this homomorphism and determine its image. We shall discuss applications to the study of character rings and character formulas.

This is a joint work with C. Hoyt.

Shmuel Zelikson

Title: On the Demazure Conjecture of Nakshima for string cones.

Abstract: Let $U_q(\mathfrak{g})$ be a quantized enveloping algebra of finite Dynkin type. The Kashiwara $B(\infty)$ crystal is a combinatorial skeleton of the negative part $U_q(\mathfrak{n}^-)$ of $U_q(\mathfrak{g})$ obtained from the canonical basis through the crystallization process. Every reduced expression of the longest element w_0 of the Weyl group induces a string (or Kashiwara) parametrization of the vertices of $B(\infty)$. Its indexing set is the set of integer points of a polyhedral cone, the string cone.

Nakashima conjectured that for some reduced expressions of w_0, one may find a set of defining inequalities for the string cone admitting a Demazure subcrystal structure. We show his conjecture to be true for reduced expressions adapted to quivers of A_n type. The ingredients of the proof are the use of Auslander-Reiten quivers and the theory of piecewise linear operators introduced by Nakashima and Zelevinsky.